Parametric and resonant transition radiation in periodic stratified structures

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(Received 16 April 2001; published 19 February 2002)

We consider the problem of electromagnetic emission when an electrically charged particle crosses a periodically stratified structure following an arbitrary linear trajectory. A theory generalizing a recently developed method [Phys. Rev. E **63**, 016613 (2001)] is presented in the framework of the classical theory of electromagnetism in continuous media. It allows one to account for both the so-called parametric radiation and the resonant transition radiation. We implement our model to interpret the experiments performed by Kaplin *et al.* [Appl. Phys. Lett. **76**, 3647 (2000)] in the x-ray domain with a stack of 300 W/B_4C bilayers irradiated by 500 MeV electrons.

DOI: 10.1103/PhysRevE.65.036501 PACS number(s): 41.60.−m

I. INTRODUCTION

When an electrically charged particle (in practice, an electron) crosses the interface separating two different materials, electromagnetic radiation is emitted, mainly in the x-ray domain. This radiation, called transition radiation (TR), is the consequence of the readjustment of the field associated with the charged particle, when it moves in a material showing a sudden change of polarization; it has been discovered by Ginsburg and Frank [1]. The emission takes place in a cone centred on the trajectory of the particle with an opening angle practically equal to $2/\gamma$, where γ is the Lorentz factor associated to the energy of the particle. Numerous teams have observed the TR by means of radiators consisting of a set of thin foils [2]. If the arrangement of foils is periodic, coherent x-ray emission, called resonant transition radiation (RTR), resulting from constructive interferences between the waves emitted by each foil can be observed [3,4] when the following resonant condition is satisfied:

$$\cos[\alpha]=\frac{1}{\beta}-p\frac{\lambda}{d}, \tag{1}$$

where α is the angle of emission with respect to the trajectory of the particle, β is the reduced speed of the particle, d the period of the arrangement, λ the wavelength of the coherent radiation, and p the order of interference. Such periodic radiators with thin films separated by vacuum are very difficult to realize for mechanical reasons. Alternatively, it has been proposed to use as radiators, periodic multilayer structures similar to x-ray multilayer interferential mirrors to produce RTR in the x-ray domain [5]; to match the angle for which the TR emission by a single interface is maximum, the period d of the multilayer structure must be in the micrometer range to produce soft x-rays and simple calculations show that in this condition, the multilayer-based RTR is a relatively intense source of x-rays [6,7]. The modelization of the RTR emission by periodic stratified structures has been done by different groups, both in a classical and quantum context [8,9,10]. More recently, a dynamical theory of electromagnetic emission by a periodic multilayer structure crossed by an electron at normal or oblique incidence [11] has shown that beside the classical RTR, an enhancement of emission occurs when conditions close to the Bragg condition are satisfied; this emission was called Bragg resonant transition radiation (BRTR). This kind of radiation is similar to the so-called parametric radiation forecasted by Dialetis [12] and evidenced by several teams in the hard x-ray domain from irradiation of natural crystals by electrons [13–16].

Standard RTR was observed by a Japanese team, which has irradiated a Ni/C multilayer structure of period d equal to 397 nm with 15 MeV electrons [17]. Their measurements are in close agreement with the values given by a rigorous classical theory of standard RTR emission from periodic stratified structures recently developed and using a matrix formalism commonly implemented in optics of multilayer media [18].

Moreover, in conditions similar to those proposed in Ref. [6], i.e., with electrons impinging the periodic multilayer structures at oblique incidence, so that the x-ray emission satisfies the Bragg condition, an relatively intense emission of hard x-ray (around 15 keV) from 500 MeV electrons crossing a W/B_4C multilayer with a period d equal to 1.24 nm has just been observed [19].

The purpose of this paper is to generalize the theory presented in Ref. [18] to the case where the incident particle enters the multilayer stack from any direction. This theory must account both for the RTR and the BRTR (or parametric radiation). Our approach is rigorous in the framework of the classical theory of electromagnetism in continuous media. In a similar way as in Ref. [18], it calls upon the matrix formalism introduced in optics to deal with the wave propagation in multilayer media; the main difference from the model presented in Ref. [18] is that we use the matrix method initially introduced by Abelès [20], which makes it possible to treat in an elegant manner, the problems of polarization.

The paper is organized as follows:

Section II is devoted to the derivation of the equation governing the electromagnetic field produced by an electron crossing two homogeneous media separated by a plane interface.

Section III introduces the matrix formalism.

Section IV presents the calculations of the field emitted from the radiator in the far zone and of the corresponding intensity in terms of energy radiated by unit frequency interval and per unit solid angle.

In Sec. V, we apply our model to account for an experiment conducted recently and reported in Ref. [19].

The conclusion and the perspectives are given in Sec. VI.

II. EQUATION OF THE ELECTROMAGNETIC FIELD PRODUCED BY AN ELECTRON CROSSING TWO HOMOGENEOUS MEDIA SEPARATED BY A PLANE INTERFACE

We consider the electromagnetic field associated with an electrically charged particle of charge q, crossing at a constant speed the interface between two homogeneous media. If $\varepsilon[\omega]$ denotes the dielectric constant and if the magnetic permeability is assumed to be equal to unity, then the Fourier transforms of the electric and magnetic field read, in the Gauss unit system,

$$\mathbf{k}\times\hat{\mathbf{H}}=-\frac{\omega\varepsilon[\omega]}{c}\hat{\mathbf{E}}-\frac{q}{c}\frac{i}{2\pi^2}\delta[\omega-\mathbf{k}\cdot\mathbf{v}]\mathbf{v}, \tag{2a}$$

$$\mathbf{k}\cdot\hat{\mathbf{E}}=-\frac{i}{2\pi^2\varepsilon[\omega]}q\,\delta[\omega-\mathbf{k}\cdot\mathbf{v}], \tag{2b}$$

$$\mathbf{k}\times\hat{\mathbf{E}}=\frac{\omega}{c}\hat{\mathbf{H}}, \tag{2c}$$

$$\mathbf{k}\cdot\hat{\mathbf{H}}=0, \tag{2d}$$

where $\mathbf{E}$, $\mathbf{H}$ are, respectively, the generic symbols for the electric field intensity and the magnetic field intensity; $\mathbf{v}$ is the speed vector of the particle with a component $\mathbf{v}_\parallel$ parallely to the interface and a component $\mathbf{v}_\perp$ along the direction perpendicular to the interface; c is the velocity of light in vacuum; ω and $\mathbf{k}$ are, respectively, the angular frequency and the wave vector of the field; δ is the Dirac distribution. In the following, we use the notation $k_0=\omega/c$.

By combining the above equations it follows that

$$\hat{\mathbf{E}}[\mathbf{k},\omega]=\frac{qi}{2\pi^2\varepsilon[\omega]}\frac{\dfrac{\omega\varepsilon[\omega]}{c^2}\mathbf{v}-\mathbf{k}}{k^2-\dfrac{\omega^2\varepsilon[\omega]}{c^2}}\delta[\omega-\mathbf{k}\cdot\mathbf{v}], \tag{3}$$

$$\hat{\mathbf{H}}[\mathbf{k},\omega]=\frac{1}{c}\frac{qi}{2\pi^2}\frac{\mathbf{k}\times\mathbf{v}}{k^2-\dfrac{\omega^2\varepsilon[\omega]}{c^2}}\delta[\omega-\mathbf{k}\cdot\mathbf{v}]. \tag{4}$$

To apply easily the boundary conditions of optics, that is, in our context, the continuity of the tangential components of the electric and magnetic field, it is convenient to introduce a partial Fourier transform of these fields; these quantities, embellished by overbar are defined by the following expressions:

$$\overline{\mathbf{E}}[\mathbf{k}_\parallel,\omega,z]=\frac{1}{(2\pi)^3}\int\int\int\mathbf{E}[\boldsymbol{\rho},z,t]e^{i(-\mathbf{k}_\parallel\cdot\boldsymbol{\rho}+\omega t)}dt\,d^2\boldsymbol{\rho}, \tag{5}$$

$$\overline{\mathbf{H}}[\mathbf{k}_\parallel,\omega,z]=\frac{1}{(2\pi)^3}\int\int\int\mathbf{H}[\boldsymbol{\rho},z,t]e^{i(-\mathbf{k}_\parallel\cdot\boldsymbol{\rho}+\omega t)}dt\,d^2\rho. \tag{6}$$

It can be shown in Appendices A and B that the above quantities are given by

$$\overline{\mathbf{E}}[\mathbf{k}_\parallel,\omega,z]=\frac{1}{v_\perp}\frac{qi}{2\pi^2\varepsilon[\omega]}\times\frac{\omega\dfrac{\varepsilon[\omega]}{c^2}\mathbf{v}_\parallel-\mathbf{k}_\parallel+\left(\omega\dfrac{\varepsilon[\omega]}{c^2}-\dfrac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp^2}\right)\mathbf{v}_\perp}{k_\parallel^2+\left(\dfrac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}\right)^2-\dfrac{\omega^2\varepsilon[\omega]}{c^2}}\times\exp\left(i\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}z\right), \tag{7}$$

and

$$\overline{\mathbf{H}}[\mathbf{k}_\parallel,\omega,z]=\frac{1}{v_\perp}\frac{qi}{2\pi^2\varepsilon[\omega]}\times\frac{\dfrac{\varepsilon[\omega]}{c}\left(\mathbf{k}_\parallel\times\mathbf{v}_\parallel+\mathbf{k}_\parallel\times\mathbf{v}_\perp+\dfrac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp^2}\mathbf{v}_\perp\times\mathbf{v}_\parallel\right)}{k^2+\left(\dfrac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}\right)^2-\dfrac{\omega^2\varepsilon[\omega]}{c^2}}\times\exp\left(i\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}z\right). \tag{8}$$

These expressions are the generalization to the oblique case of Eqs. (12), (13) of Ref. [18]. The general solution of the problem is obtained as the sum of the particular solution of the inhomogeneous Maxwell equations and the solution of the homogeneous equations. The latter is, in terms of electric field intensity,

$$\mathbf{E}_0[\boldsymbol{\rho},z,t]=\int\int\int\tilde{\mathbf{E}}_0[\mathbf{k}_\parallel,k_\perp]e^{i(\mathbf{k}_\parallel\cdot\boldsymbol{\rho}+k_\perp z-\omega t)}d^2\mathbf{k}_\parallel dk_\perp, \tag{9}$$

where ω, $\mathbf{k}_\parallel$, and $k_\perp$ are related by the dispersion relation:

$$k_\perp^2=\frac{\omega^2}{c^2}\varepsilon[\omega]-\mathbf{k}_\parallel^2. \tag{10}$$

As shown previously, one introduces the partial Fourier transforms

$$\overline{\mathbf{E}}_0[\mathbf{k}_\parallel,\omega,z]=\frac{1}{(2\pi)^3}\int\int\int\mathbf{E}_0[\boldsymbol{\rho},z,t]e^{i(-\mathbf{k}_\parallel\cdot\boldsymbol{\rho}+\omega t)}dt\,d^2\boldsymbol{\rho}, \tag{11}$$

and

$$\overline{\mathbf{H}}_0[\mathbf{k}_\parallel,\omega,z]=\frac{1}{(2\pi)^3}\int\int\int \mathbf{H}_0[\boldsymbol{\rho},z,t]e^{i(-\mathbf{k}_\parallel\cdot\boldsymbol{\rho}+\omega t)}dt\,d^2\boldsymbol{\rho}. \tag{12}$$

Then one obtains from calculations similar to the ones of Appendices A and B:

$$\overline{\mathbf{E}}_0[\mathbf{k}_\parallel,\omega,z]=\int\int\int \tilde{\mathbf{E}}_0[\mathbf{k}']e^{ik'_\perp z}\delta[\mathbf{k}'_\parallel-\mathbf{k}_\parallel]\times\delta[\omega'-\omega]d^2\mathbf{k}'_\parallel dk'_\perp, \tag{13}$$

and similar results for the magnetic field.
After some manipulations, one gets

$$\overline{\mathbf{E}}_0[\mathbf{k}_\parallel,\omega,z]=\overline{\mathbf{E}}'_{0t}[\mathbf{k}_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{E}}'_{0r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{14}$$

$$\overline{\mathbf{H}}_0[\mathbf{k}_\parallel,\omega,z]=\overline{\mathbf{H}}'_{0t}[k_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{H}}'_{0r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{15}$$

where one has

$$\overline{\mathbf{E}}'_{0t}[\mathbf{k}_\parallel,\omega]=\frac{1}{2k_\perp[k_\parallel,\omega]c^2}\left(2\omega\varepsilon[\omega]+\omega^2\frac{d\varepsilon[\omega]}{d\omega}\right)\tilde{\mathbf{E}}_{0t}[\mathbf{k}_\parallel,\omega], \tag{16}$$

and similar expressions for $\overline{\mathbf{E}}'_{0r}[\mathbf{k}_\parallel,\omega]$, $\overline{\mathbf{H}}'_{0t}[k_\parallel,\omega]$, and $\overline{\mathbf{H}}'_{0r}[\mathbf{k}_\parallel,\omega]$.

The boundary conditions state that the tangential components of the electric and magnetic field intensities must be continuous at each interface.
If one denotes by

$$\overline{\mathbf{E}}_{0\parallel}[\mathbf{k}_\parallel,\omega,z]=\overline{\mathbf{E}}'_{0\parallel t}[\mathbf{k}_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{E}}'_{0\parallel r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{17}$$

and

$$\overline{\mathbf{H}}_{0\parallel}[\mathbf{k}_\parallel,\omega,z]=\overline{\mathbf{H}}'_{0\parallel t}[\mathbf{k}_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{H}}'_{0\parallel r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{18}$$

the tangential components of the electric and magnetic fields associated to the homogeneous equation, the continuity conditions require that the following quantities:

$$\frac{1}{v_\perp}\frac{qi}{2\pi^2\varepsilon[\omega]}\frac{\omega\frac{\varepsilon[\omega]}{c^2}\mathbf{v}_\parallel-\mathbf{k}_\parallel}{k_\parallel^2+\left(\frac{\omega-k_\parallel\cdot v_\parallel}{v_\perp}\right)^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}e^{i\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}z}+\overline{\mathbf{E}}'_{0\parallel t}[\mathbf{k}_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{E}}'_{0\parallel r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{19}$$

and

$$\frac{1}{cv_\perp}\frac{qi}{2\pi^2}\frac{\mathbf{k}_\parallel\times\mathbf{v}_\perp+\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp^2}\mathbf{v}_\perp\times\mathbf{v}_\parallel}{k^2+\left(\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp}\right)^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}\exp\left(i\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_\perp z}\right)+\overline{\mathbf{H}}'_{0\parallel t}[\mathbf{k}_\parallel,\omega]e^{ik_\perp[k_\parallel,\omega]z}+\overline{\mathbf{H}}'_{0\parallel r}[\mathbf{k}_\parallel,\omega]e^{-ik_\perp[k_\parallel,\omega]z}, \tag{20}$$

are conserved at the interfaces.

III. MATRIX FORMALISM FOR THE PROPAGATION OF THE FIELD IN A PERIODIC STACK OF PLANE LAYERS

One consider a laboratory reference system (x, y, z), for which the interfaces between the layers are parallel to the plane (x, y); the z axis is orthogonal to the stratification planes (x, y). In fact, the relevant system of reference (X, Y, Z) that makes it possible to introduce conveniently the amplitudes of the different waves will be called the canonical system. In this system, which depends on the tangential component of the wave vector, the tangential component of the field has only a Y component, the X component being null. This system of reference can be obtained as follows:

The unit vector of the Z axis $\mathbf{u}_z$ is along the direction normal to the stratification planes, that is, along the z axis.

The unit vector of the Y axis $\mathbf{u}_y$ is colinear with the tangential component of the wave vector $\mathbf{k}$.

The unit vector of the X axis $\mathbf{u}_x$ is obtained from the cross product $\mathbf{u}_x=\mathbf{u}_y\times\mathbf{u}_z$.

In this canonical system, an electromagnetic plane wave can be decomposed in a transverse magnetic (TM) wave and a transverse electric (TE) wave. For the TM wave, the magnetic field has only a component along the X axis and the electric field can be determined from the magnetic field by means of a Maxwell equation. For the TE wave, it is the electric field that has only a component along the X axis, and the magnetic field can be calculated by means of a Maxwell equation. The waves that propagate with a given tangential wave vector $\mathbf{k}_\parallel$ can be split into a transmitted TM component, a reflected TM component, a transmitted TE component, and a reflected TE component, which can be presented in the form of a quadrivector

$$\mathbf{TR}[\mathbf{k}_\parallel,\omega,z]=\begin{pmatrix} T_{\mathrm{TM}}[\mathbf{k}_\parallel,\omega,z]\\ R_{\mathrm{TM}}[\mathbf{k}_\parallel,\omega,z]\\ T_{\mathrm{TE}}[\mathbf{k}_\parallel,\omega,z]\\ R_{\mathrm{TE}}[\mathbf{k}_\parallel,\omega,z]\end{pmatrix}. \tag{21}$$

In the following, we will be led to use the tangential components of the homogeneous electromagnetic field and to express them in the form of a quadrivector in the canonical reference system:

$$\mathbf{F}[\mathbf{k}_\parallel,\omega,z]=\begin{pmatrix}\overline{H}_{0x}[\mathbf{k}_\parallel,\omega,z]\\ \overline{E}_{0y}[\mathbf{k}_\parallel,\omega,z]\\ \overline{E}_{0x}[\mathbf{k}_\parallel,\omega,z]\\ \overline{H}_{0y}[\mathbf{k}_\parallel,\omega,z]\end{pmatrix}. \tag{22}$$

The matrix that transforms the quadrivector **TR** into the quadrivector **F**, that is, $\mathbf{F}=M\ \mathbf{TR}$, is given in the canonical system by

$$M=\begin{pmatrix}1 & 1 & 0 & 0\\ -\dfrac{k_\perp}{k_0} & \dfrac{k_\perp}{k_0} & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & \dfrac{k_\perp}{k_0} & -\dfrac{k_\perp}{k_0}\end{pmatrix}. \tag{23}$$

In the canonical system, the total electromagnetic field (i.e., inhomogeneous field plus homogeneous field) can be totally known from a quadrivector **Q** built by the means of the corresponding tangential components of the electric and magnetic fields,

$$\mathbf{Q}[\mathbf{k}_\parallel,\omega,z]=\begin{pmatrix}\overline{\mathbf{H}}_x[\mathbf{k}_\parallel,\omega,z]\\ \overline{\mathbf{E}}_y[\mathbf{k}_\parallel,\omega,z]\\ \overline{\mathbf{E}}_x[\mathbf{k}_\parallel,\omega,z]\\ \overline{\mathbf{H}}_y[\mathbf{k}_\parallel,\omega,z]\end{pmatrix}. \tag{24}$$

From the results of Sec. II, it follows that the **Q** quadrivector reads

$$\mathbf{Q}[\mathbf{k}_\parallel,\omega,z]=\mathbf{S}[\mathbf{k}_\parallel,\omega]e^{ik_z z}+\mathbf{F}[\mathbf{k}_\parallel,\omega,z], \tag{25}$$

with

$$k_z=\frac{\omega-\mathbf{k}_\parallel\cdot\mathbf{v}_\parallel}{v_z}. \tag{26}$$

The quadrivector **S** can be expressed in the canonical system as

$$\mathbf{S}=\frac{iq}{2\pi^2\varepsilon v_z\left(\left(\dfrac{\omega-k_y v_y}{v_z}\right)^2-k_z{}^2\right)}\times\begin{pmatrix}\dfrac{\varepsilon}{c}k_y v_z-\dfrac{v_y(\omega-k_y v_y)}{v_z}\\ \dfrac{\varepsilon}{c^2}\omega v_y-k_y\\ \dfrac{\varepsilon}{c^2}\omega v_x\\ \dfrac{\varepsilon}{c}\dfrac{v_x(\omega-k_y v_y)}{v_z}\end{pmatrix}\exp\left[i\frac{z(\omega-k_y v_y)}{v_z}\right]. \tag{27}$$

The stack consists of an arrangement of plane layers characterized by their dielectric constants ε_j and their thicknesses d_j. At the interface, whose abscissa is z_{j-1} and which separates the layer $j-1$ from the layer j, the continuity relations give the set of recurrent equations,

$$\mathbf{F}_{j-1}[\mathbf{k}_\parallel,\omega,z_{j-1}]+\mathbf{S}_{j-1}[\mathbf{k}_\parallel,\omega]e^{ik_z z_{j-1}}=\mathbf{F}_j[\mathbf{k}_\parallel,\omega,z_{j-1}]+\mathbf{S}_j[\mathbf{k}_\parallel,\omega]e^{ik_z z_{j-1}}. \tag{28}$$

By introducing the following quantities,

$$\Delta_{j-1}=\mathbf{S}_{j-1}-\mathbf{S}_j, \tag{29}$$

one has

$$\mathbf{F}_j[\mathbf{k}_\parallel,\omega,z_{j-1}]=\mathbf{F}_{j-1}[\mathbf{k}_\parallel,\omega,z_{j-1}]+\Delta_{j-1}[\mathbf{k}_\parallel,\omega]e^{ik_z z_{j-1}}. \tag{30}$$

The $\mathbf{F}_{j-1}[\mathbf{k}_\parallel,\omega,z_{j-1}]$ can be deduced from $\mathbf{F}_{j-1}[\mathbf{k}_\parallel,\omega,z_{j-2}]$ by means of the Abelès formalism [20,21]:

$$\mathbf{F}_j[\mathbf{k}_\parallel,\omega,z_{j-1}]=A[\mathbf{k}_\parallel,\omega,z_{j-1}-z_{j-2}]\cdot\mathbf{F}_{j-1}[\mathbf{k}_\parallel,\omega,z_{j-2}]+\Delta_{j-1}[\mathbf{k}_\parallel,\omega]e^{ik_z z_{j-1}}, \tag{31}$$

where $A[\mathbf{k}_\parallel,\omega,z_{j-1}-z_{j-2}]$ is the Abelès matrix for a single layer. In the canonical system, this matrix takes the following simple form:

$$A[\mathbf{k}_\parallel,\omega,d_j]=\begin{pmatrix}\cos[k_\perp d_j] & -\dfrac{i\varepsilon k}{k_\perp}\sin[k_\perp d_j] & 0 & 0\\ -\dfrac{ik_\perp}{\varepsilon k}\sin[k_\perp d_j] & \cos[k_\perp d_j] & 0 & 0\\ 0 & 0 & \cos[k_\perp d_j] & \dfrac{ik}{k_\perp}\sin[k_\perp d_j]\\ 0 & 0 & \dfrac{ik_\perp}{k}\sin[k_\perp d_j] & \cos[k_\perp d_j]\end{pmatrix}. \tag{32}$$

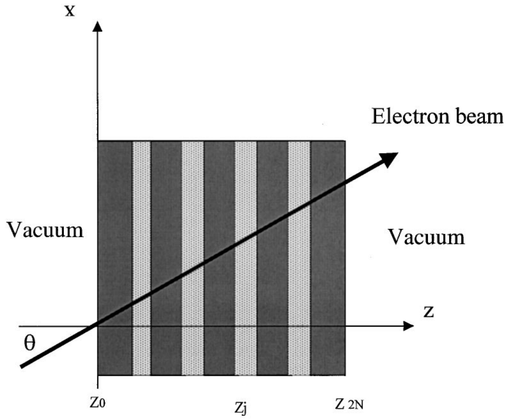

FIG. 1. Scheme of a periodic multilayer stack made up of alternate layers of a material of dielectric constant ε_1 and thickness d_1 and of a material of dielectric constant ε_2 and thickness d_2. The number of bilayers is N. The electron enters the stack with an incident angle θ.

Now, one handles the case of a periodic multilayer stack immersed in the vacuum (see Fig. 1). We suppose that the stack consists of an alternate arrangement of two materials characterized by the dielectric constants ε_1 and ε_2 and the thicknesses d_1 and d_2, respectively. We denote by d, the period d_1+d_2, and by N, the number of periods. The number of the first medium is 0 while the one of the last medium is equal to $2N+1$.
Using twice, the Eq. (30), one obtains,

$$\mathbf{F}'_{2j+1}[\mathbf{k}_\parallel,\omega,z_{2j}]=\mu[\mathbf{k}_\parallel,\omega,d_1,d_2]\cdot\mathbf{F}'_{2j-1}[\mathbf{k}_\parallel,\omega,z_{2j-2}]+\delta[\mathbf{k}_\parallel,\omega,d_1,d_2,k_z], \tag{33}$$

with

$$\mathbf{F}'_{2j+1}=e^{-ik_z z_{2j+1}}\mathbf{F}_{2j+1}[\mathbf{k}_\parallel,\omega,z_{2j}], \tag{34}$$

$$\mu[\mathbf{k}_\parallel,\omega,d_1,d_2]=e^{-ik_z(d_1+d_2)}A[\mathbf{k}_\parallel,\omega,d_2]\cdot A[\mathbf{k}_\parallel,\omega,d_1], \tag{35}$$

and

$$\delta[\mathbf{k}_\parallel,\omega,d_1,d_2,k_z]=A[\mathbf{k}_\parallel,\omega,d_2]\cdot\Delta[\mathbf{k}_\parallel,\omega]e^{-ik_z(d_1+d_2)}-\Delta[\mathbf{k}_\parallel,\omega]e^{ik_z d_2}, \tag{36}$$

where

$$\Delta=\Delta_1. \tag{37}$$

In this derivation, we have used the fact that $\Delta_{n+1}=-\Delta_n$. A little algebra leads to a linear relationship between $\mathbf{F}'_{2j+1}$ and $\mathbf{F}'_1$:

$$\mathbf{F}'_{2j+1}=S_j[\mu]\delta+P_j[\mu]\mathbf{F}'_1, \tag{38}$$

where

$$S_j[\mu]=\sum_{i=0}^{j-1}\mu^i \quad \text{and} \quad P_j[\mu]=\mu^j. \tag{39}$$

IV. DETERMINATION OF THE RADIATION INTENSITY IN THE FAR ZONE

The electric field $\mathbf{E}_D[\mathbf{D},t]$ seen by a detector located at $\mathbf{D}=(\rho,D)$ in the laboratory frame can be calculated by Fourier transform

$$\mathbf{E}_D[\mathbf{D},t]=\int\int d^2\mathbf{k}_\parallel\int d\omega\,\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_\parallel,\omega]\times e^{i\eta k_\perp[k_\parallel,\omega]z_B}e^{i(k_\parallel\cdot\boldsymbol{\rho}+k_\perp[k_\parallel,\omega]D-\omega t)}\frac{\partial k_\perp[k_\parallel,\omega]}{\partial\omega}, \tag{40}$$

where $\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_\parallel,\omega]$ is the partial Fourier transform of the electric field calculated at the appropriate boundary situate at z_B. The presence of the symbol η is due to the fact that dispersion relation [Eq. (10)] admits the two solutions $+k_\perp$ and $-k_\perp$.

In fact, we shall be interested in the time Fourier transform for the calculation of the radiated intensity,

$$\hat{\mathbf{E}}_D[\mathbf{D},\omega']=\frac{1}{2\pi}\int dt\,e^{i\omega' t}\int\int d^2\mathbf{k}_\parallel\int d\omega\,\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_\parallel,\omega]\times e^{i\eta k_\perp[k_\parallel,\omega]z_B}e^{i(k_\parallel\cdot\boldsymbol{\rho}+k_\perp[k_\parallel,\omega]D-\omega t)}\frac{\partial k_\perp[k_\parallel,\omega]}{\partial\omega}. \tag{41}$$

The integrations over t and ω give

$$\hat{\mathbf{E}}_D[\mathbf{D},\omega']=\int\int d^2\mathbf{k}_\parallel\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_\parallel,\omega']e^{i\eta k_\perp[k_\parallel,\omega']z_B}\times e^{i(k_\parallel\cdot\boldsymbol{\rho}+k_\perp[k_\parallel,\omega']D)}\frac{\partial k_\perp[k_\parallel,\omega']}{\partial\omega'}. \tag{42}$$

Now we calculate the field at an observation point $\mathbf{D}$ situated in far zone, the spherical coordinates of which are D, α, φ; note that the reference frame is chosen so that $\varphi=90°$, when the observation point $\mathbf{D}$ is located in the incident plane formed by the speed vector and the normal to the stratification planes. Provided that the values of D's are large enough, the integrals (42) can be performed by using the method of stationary phase generalized at two dimensions [22]. The calculation gives

$$\hat{\mathbf{E}}_D[\mathbf{D},\omega]=\sigma 2\pi i\frac{\omega}{c}\cos[\alpha]\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_{\parallel S},\omega]e^{i\eta k_\perp[\mathbf{k}_{\parallel S},\omega]z_B}\times\frac{e^{iD\phi[\mathbf{k}_{\parallel S}]}}{D}\frac{\partial k_\perp[k_{\parallel S},\omega]}{\partial\omega}, \tag{43}$$

where ϕ is the so-called stationary phase and $\mathbf{k}_{\parallel S}$ is the stationary wave vector given in the canonical system by

$$\mathbf{k}_{\parallel S}=k_0\begin{pmatrix}0\\ \sin[\alpha]\\ 0\end{pmatrix}, \tag{44}$$

σ is an irrelevant quantity that depends on the geometry and whose modulus is equal to unity. The problem at this stage is to calculate the electric field at the boundaries $\tilde{\mathbf{E}}_{B\eta}[\mathbf{k}_{\parallel S},\omega]$. To do it, one needs the homogeneous field quadrivectors $\mathbf{F}$'s at the two boundaries ($z_B=z_0=0$ and $z_B=z_{2N}$) of the stack. According to Eq. (38), they are connected by the following vector equation:

$$\mathbf{F}'[\mathbf{k}_{\parallel S},\omega,z_{2N}]=S_N[\mu]\cdot\delta[\mathbf{k}_{\parallel S},\omega]+P_N[\mu]\cdot\mathbf{F}'[\mathbf{k}_{\parallel S},\omega,z_0]. \tag{45}$$

Moreover, at the two boundaries, these homogeneous field quadrivectors $\mathbf{F}$'s are related to the corresponding amplitude quadrivectors $\mathbf{TR}[\mathbf{k}_\parallel,\omega,z_0]$ and $\mathbf{TR}[\mathbf{k}_\parallel,\omega,z_{2N}]$ by means of the matrix M, which in the canonical representation, is given by Eq. (22).

To solve the problem, one has to take into account the fact that there are no incoming waves in the extreme media, so that the Eq. (45) becomes:

$$\mathbf{F}'_o=S_N[\mu]\cdot\delta[\mathbf{k}_{\parallel S},\omega]+P_N[\mu]\cdot\mathbf{F}'_i, \tag{46}$$

with

$$\mathbf{F}'_o=\exp[-ik_zd]M\begin{pmatrix}T_o[\mathrm{TM}]\\0\\T_o[\mathrm{TE}]\\0\end{pmatrix}. \tag{47}$$

and

$$\mathbf{F}'_i=\exp[-ik_zd]M\begin{pmatrix}0\\R_i[\mathrm{TM}]\\0\\R_i[\mathrm{TE}]\end{pmatrix}. \tag{48}$$

$T_o[\mathrm{TM}]$ and $T_o[\mathrm{TE}]$ stand for the amplitude of the transmitted TM and TE waves at $z=z_{2N}$, respectively, while $R_i[\mathrm{TM}]$ and $R_i[\mathrm{TE}]$ stand for the amplitude of the reflected TM and TE waves at $z=z_o$, respectively: the suffix "o" is for outgoing while the suffix "i" is for incoming in relation with the direction of propagation of the electrons with respect to the stack.

The resolution of the system (46) gives the values of $T_o[\mathrm{TM}]$, $T_o[\mathrm{TE}]$, $R_i[\mathrm{TM}]$, and $R_i[\mathrm{TE}]$ from which one can calculate the reflected and transmitted intensities of the radiated far-field per unit angular frequency $d\omega$ and per unit solid angle $d\Omega$ according to their different polarizations, by using the relationship

$$\frac{\partial^3 I[\Omega,\omega]}{\partial\omega\,\partial^2\Omega}=cD^2\|\hat{\mathbf{E}}_D[\mathbf{D},\omega]\|^2. \tag{49}$$

V. APPLICATIONS

First, we have checked that our generalized approach is able to retrieve the results obtained in the case of RTR with the previous model [18]. RTR occurs when one of the quantities S_N involved in Eq. (45) presents a resonant behavior: there are constructive interferences between the RT emitted by each bilayer. This resonant condition reduces to the classical resonant condition [cf. Eq. (1)] for the RTR when the indices of refraction are small enough; this case corresponds to the so-called x-ray regime studied in Ref. [11].

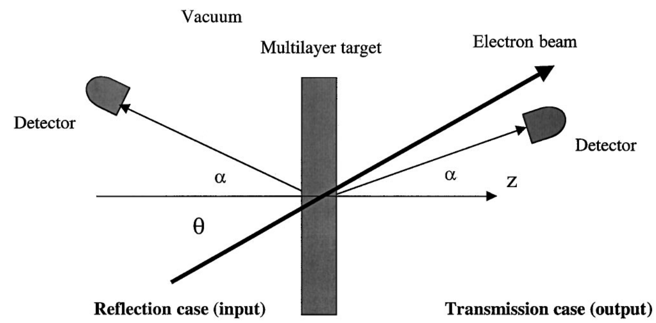

FIG. 2. Geometry for the observation of the emitted radiation.

The present generalized model was, in fact, developed to account for the so-called Bragg resonant transition radiation or, to use a more common vocable, the parametric radiation(PR).

We consider the experiment recently performed in the x-ray domain with 500 MeV electrons impinging at oblique incidence, a stack of 300 W/B_4C bilayers [19]. The geometry was choosen to observe x-rays around 15 keV (Fig. 2). Figure 3 shows the intensity in terms of number of photons/electron/eV/sr calculated versus the photon energy within our model, corresponding to this experiment; the azimuthal angle of observation φ retained for the calculation is equal to 90° (it means that the observation is done in the incident plane formed by the electron speed vector and the normal to the stratification planes). The intensity is calculated for the TM polarization since the contribution of the TE polarization vanishes for this value of the azimutal angle because of symmetry reasons. The spectrally integrated intensity calculated from our model is about 100 photons/electron/sr, while the experimental value is about 0.22 photons/electron/sr. The authors of the paper reporting the experimental data [19] claim that this latter value is in agreement with a calculation based

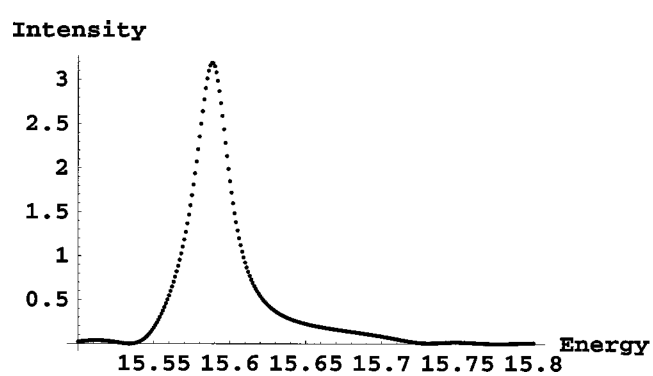

FIG. 3. Calculated TM spectrum of the parametric radiation emitted by a target consisting of 300 W B_4C bilayers crossed by a 500 MeV electron. The thickness of the W layer is 0.5 nm and the thickness of the B_4C layer is 0.7 nm. The incident angle of the electron is 88°. The spherical angles of the observation point are: $\alpha=1.9°$ and $\varphi=90°$.

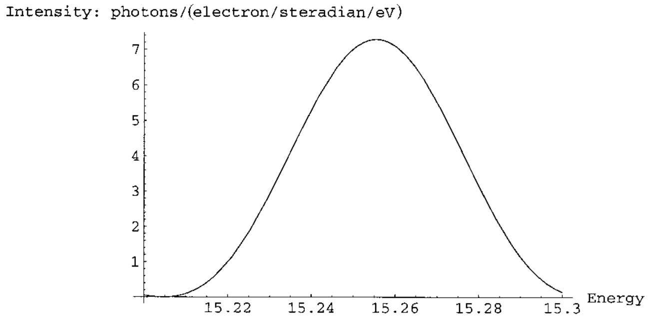

FIG. 4. Spectrum of the parametric radiation emitted by the same target as in Fig. 3, calculated from the virtual-quanta model presented in Ref. [11].

on a simple model of virtual-photon reflection. Unfortunately, no reference concerning this model is given. A theoretical approach connected to the method of virtual quanta, that we have previously developed [11], gives a spectrally integrated intensity in agreement with the present theory (See Fig. 4). We have observed that the calculated intensity is very sensitive to the value of the observation angle α: a shift of 10^{-1} degree on this value with respect to the optimal α value leads to a fall in the intensity by a factor about 100. The fact could explain the discrepancy between theory and experiment. Moreover, the multilayer structure is likely not ideal (interfacial roughness, small irregularities in the stack), so that these imperfections lead, in practice, to a reduction of the emitted intensity. In Fig. 5, we have plotted for the optimal energy (about 15.58 keV), the intensity versus the azimuthal angle φ: the difference in the spatial distribution for the TM and TE cases is clearly evidenced.

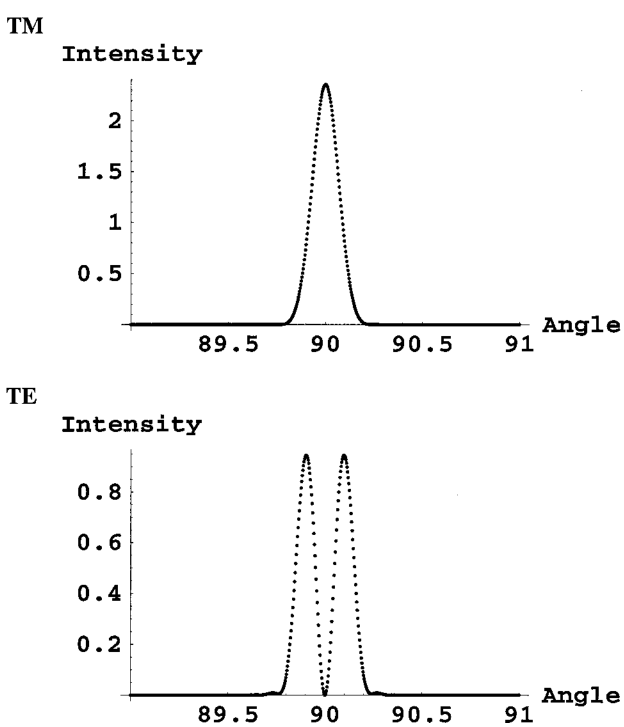

FIG. 5. Azimutal angular (φ) distribution of the parametric radiation emitted by the same target as in Fig. 3. The calculation is done for the TM and TE polarizations at the optimum energy (15.58 keV).

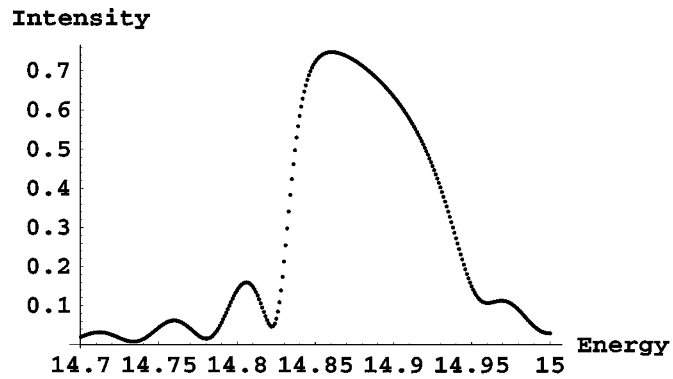

FIG. 6. TM reflectivity versus the photon energy of the multilayer interferential mirror equivalent to the target used for the calculation of Figs. 3, 4, and 5. The incident angle of the x rays is 88°. The α observation angle is equal to 2°.

Parametric radiation occurs when the periodic multilayer stack is in the Bragg regime for the emitted radiation. The observation angle for which the PR emission is maximum does not follow the Snell law, which means that the optimal value α_{opt} of the angle α is not equal to $\pi/2$-θ (that is, 2° in the case of interest), but to 1.9°; this latter value is exactly the value of the position of the detector reported in Ref. [19]. Note that we have observed that the value of the optimum energy depends drastically on the electron energy. Figure 5 shows the TM reflectivity of the multilayer mirror corresponding to the multilayer target, for an incident angle θ equal to 88°. One sees the shift between the Bragg energy (around 14.85 keV) and the optimal photon energy of the TR emission (15.58 keV). It is interesting to note that the theoretical spectral bandwidth (SBW) of the Bragg reflection (about 56 eV) as shown in Fig. 6 is considerably broader than the theoretical SBW of the PR emission (about 29 eV).

VI. CONCLUSIONS AND PERSPECTIVES

We have developed a rigorous theory, in the framework of the classical theory of electromagnetism in continuous media, which modelizes both the RTR and the PR emission from periodically stratified targets. The model makes it possible to account for the main results of recent RTR and PR experiments [17,19].

In a forthcoming paper, this model will be systematically used to study the influence of the different parameters on the PR emission; the polarization of the radiation versus the observation direction will be especially examined. The influence of the electron energy, of the number of bilayers, and of the interfacial roughness will be also studied. Careful experimental investigations would be useful to check the validity of our model.

APPENDIX A

The transverse spatial and temporal Fourier transform of the field is defined by

$$\overline{\mathbf{E}}(\mathbf{k}_\parallel',\omega',z)=\frac{1}{(2\pi)^3}\int\int\int \mathbf{E}(\boldsymbol{\rho},z,t)e^{i(-\mathbf{k}_\parallel'\cdot\boldsymbol{\rho}+\omega' t)}dt\,d^2\boldsymbol{\rho}. \tag{A1}$$

Taking into account Eq. (3) yields

$$\overline{\mathbf{E}}(\mathbf{k}_\parallel',\omega',z)=\frac{1}{(2\pi)^3}\int\int\int\int\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega]}\times\frac{\frac{\omega\varepsilon[\omega]}{c^2}\mathbf{v}-\mathbf{k}}{k^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}\delta[\omega-\mathbf{k}\cdot\mathbf{v}]e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}d\omega\,d^3\mathbf{k}\times e^{i(-\mathbf{k}_\parallel'\cdot\boldsymbol{\rho}+\omega' t)}dt\,d^2\boldsymbol{\rho}. \tag{A2}$$

Performing the integration over the time t and the variable ρ gives

$$\overline{\mathbf{E}}(\mathbf{k}_\parallel',\omega',z)=\int\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega]}\frac{\frac{\omega\varepsilon[\omega]}{c^2}\mathbf{v}-\mathbf{k}}{k^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}\times\delta[\omega-\mathbf{k}\cdot\mathbf{v}]e^{ik_\perp z}\delta[\mathbf{k}_\parallel-\mathbf{k}_\parallel']\times\delta[\omega'-\omega]d\omega\,d^2k_\parallel dk_\perp. \tag{A3}$$

The integration over ω leads to

$$\overline{\mathbf{E}}(\mathbf{k}_\parallel',\omega',z)=\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega']}\frac{\frac{\omega'\varepsilon[\omega']}{c^2}\mathbf{v}-\mathbf{k}}{k^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\delta[\omega'-\mathbf{k}\cdot\mathbf{v}]e^{ik_\perp z}\delta(\mathbf{k}_\parallel-\mathbf{k}_\parallel')d^2\mathbf{k}_\parallel dk_\perp. \tag{A4}$$

The integration over $\mathbf{k}_\parallel$ gives,

$$\overline{\mathbf{E}}(\mathbf{k}_\parallel',\omega',z)=\int \frac{qi}{2\pi^2\varepsilon[\omega']}\frac{\frac{\omega'\varepsilon[\omega']}{c^2}\mathbf{v}-\mathbf{k}_\parallel'-\mathbf{k}_\perp}{k_\parallel'^2+k_\perp^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\delta(\omega'-\mathbf{k}_\parallel'\cdot\mathbf{v}_\parallel-\mathbf{k}_\perp\cdot\mathbf{v}_\perp)e^{ik_\perp z}dk_\perp. \tag{A5}$$

After a last integration over $k_\perp$, ones obtains the expression of $\overline{\mathbf{E}}_\parallel$,

$$\overline{\mathbf{E}}_\parallel(\mathbf{k}_\parallel',\omega',z)=\frac{1}{v_\perp}\frac{qi}{2\pi^2\varepsilon[\omega']}\times\frac{\frac{\omega'\varepsilon[\omega']}{c^2}\mathbf{v}_\parallel-\mathbf{k}_\parallel'}{k_\parallel'^2+\left(\frac{\omega'-\mathbf{k}_\parallel'\cdot\mathbf{v}_\parallel}{v_\perp}\right)^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\exp\left(i\frac{\omega'-\mathbf{k}_\parallel'\cdot\mathbf{v}_\parallel}{v_\perp}z\right). \tag{A6}$$

APPENDIX B

The transverse spatial and temporal Fourier transform of the magnetic field is defined by

$$\overline{\mathbf{H}}(\mathbf{k}_\parallel',\omega',z)=\frac{1}{(2\pi)^3}\int\int\int \mathbf{H}(\rho,z,t)e^{i(-\mathbf{k}_\parallel'\cdot\boldsymbol{\rho}+\omega' t)}dt\,d^3\boldsymbol{\rho}. \tag{B1}$$

Taking into account Eq. (4) yields

$$\overline{\mathbf{H}}(\mathbf{k}_\parallel',\omega',z)=\frac{1}{(2\pi)^3}\int\int\int\int\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega]}\frac{\frac{\varepsilon[\omega]}{c}\mathbf{k}\times\mathbf{v}}{k^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}\times\delta[\omega-\mathbf{k}\cdot\mathbf{v}]e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}d\omega\,d^3\mathbf{k}e^{i(-\mathbf{k}_\parallel'\cdot\boldsymbol{\rho}+\omega' t)}dt\,d^2\boldsymbol{\rho}. \tag{B2}$$

Performing the integration over the time t and the variable $\boldsymbol{\rho}$ gives

$$\overline{\mathbf{H}}(\mathbf{k}_\parallel',\omega',z)=\int\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega]}\frac{\frac{\varepsilon[\omega]}{c}\mathbf{k}\times\mathbf{v}}{k^2-\frac{\omega^2\varepsilon[\omega]}{c^2}}\times\delta[\omega-\mathbf{k}\cdot\mathbf{v}]e^{ik_\perp z}\delta[\mathbf{k}_\parallel-\mathbf{k}_\parallel']\delta[\omega'-\omega]d\omega\,d^2k_\parallel dk_\perp. \tag{B3}$$

The integration over ω leads to

$$\overline{\mathbf{H}}(\mathbf{k}_\parallel',\omega',z)=\int\int\int \frac{qi}{2\pi^2\varepsilon[\omega']}\frac{\frac{\varepsilon[\omega']}{c}\mathbf{k}\times\mathbf{v}}{k^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\delta[\omega'-\mathbf{k}\cdot\mathbf{v}]e^{ik_\perp z}\delta[\mathbf{k}_\parallel-\mathbf{k}_\parallel']d^2\mathbf{k}_\parallel dk_\perp. \tag{B4}$$

The integration over $\mathbf{k}_\parallel$ gives,

$$\overline{\mathbf{H}}(\mathbf{k}_{\parallel}',\omega',z)=\int\frac{qi}{2\pi^2\varepsilon[\omega']}\frac{\frac{\varepsilon[\omega']}{c}(\mathbf{k}_{\parallel}'+\mathbf{k}_{\perp})\times\mathbf{v}}{k_{\parallel}'^2+k_{\perp}^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\delta[\omega'-\mathbf{k}_{\parallel}'\cdot\mathbf{v}_{\parallel}-\mathbf{k}_{\perp}\cdot\mathbf{v}_{\perp}]e^{ik_{\perp}z}dk_{\perp}. \tag{B5}$$

After a last integration over $k_{\perp}$ one obtains the following expression for $\overline{\mathbf{H}}_{\parallel}$:

$$\overline{\mathbf{H}}_{\parallel}(\mathbf{k}_{\parallel}',\omega',z)=\frac{1}{v_{\perp}}\frac{qi}{2\pi^2\varepsilon[\omega']}\times\frac{\frac{\varepsilon[\omega']}{c}\left(\mathbf{k}_{\parallel}'\times\mathbf{v}_{\perp}+\frac{\omega'-\mathbf{k}_{\parallel}'\cdot\mathbf{v}_{\parallel}}{v_{\perp}^2}\mathbf{v}_{\perp}\times\mathbf{v}_{\parallel}\right)}{k_{\parallel}'^2+\left(\frac{\omega'-\mathbf{k}_{\parallel}'\cdot\mathbf{v}_{\parallel}}{v_{\perp}}\right)^2-\frac{\omega'^2\varepsilon[\omega']}{c^2}}\times\exp\left(i\frac{\omega'-\mathbf{k}_{\parallel}'\cdot\mathbf{v}_{\parallel}}{v_{\perp}}z\right). \tag{B6}$$

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